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Spontaneous breaking of the lepton number and invisible majoron in a 3-3-1 model

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Abstract. In this work we implement the spontaneous breaking of lepton number in version II of the 3-3-1 models and study their phenomenological consequences. The main result of this work is that our majoron is invisible even though it belongs to a triplet representation by the 3-3-1 symmetry.

1 Introduction

There is already a vast literature concerning the class of models with gauge structure, $SU(3)_C \times SU(3)_{\mathrm{L}} \times U(1)_N$, (3-3-1) [1–5]. In these models the anomaly cancellation requires a minimum of three families (or a multiple of three in larger versions). Besides, there is a bunch of new particles and interactions which make these models phenomenologically rich and attractive as an alternative to the standard model (SM). However if we assume that in the realm of intermediate energy there are no exotic leptons, then the 3-3-1 symmetry allows for only two possible gauge models for the strong and electroweak interactions. We refer to such models as version I and version II. The version I is the one suggested by Pisano–Pleitez and Frampton [1]. In this version the standard leptons compose the following triplet: $(\nu_{\mathrm{L}}, l_{\mathrm{L}}, l^{\mathrm{c}}_{\mathrm{R}})^{\mathrm{T}}$. The version II is the 3-3-1 model with right-handed neutrinos [2]. In it the standard leptons constitute the following triplet: $(\nu_{\mathrm{L}}, l_{\mathrm{L}}, \nu^{\mathrm{c}}_{\mathrm{R}})^{\mathrm{T}}$.

One of the peculiar aspects of 3-3-1 models is that the Peccei–Quinn (PQ) [6] and the lepton number symmetries emerge naturally in both versions and their scalar sector provides a simple implementation of the spontaneous breaking of such symmetries [7,8].

Despite the fact that lepton symmetry is of great interest for particle physics, since its violation is a necessary condition to generate Majorana mass term for neutrinos, it was scarcely developed in both versions of the 3-3-1 models [8]. For example, as regards lepton symmetry, an explicit implementation of its spontaneous breaking is missing in version II of the 3-3-1 models. On the other side, PQ symmetry has received great attention in both versions [9,10]

In view of this, in this work we implement the spontaneous breaking of lepton number symmetry and discuss some of their consequences in the version II of the 3-3-1 models.

This work is organized as follows. In Sect. 2 we present the particle content of the model. Next, in Sect. 3 we implement the spontaneous breaking of the lepton number and identify the majoron. In Sect. 4 we concentrate on the phenomenology of our majoron. In Sect. 5 we discuss neutrino masses. Finally, in Sect. 6, we present our conclusions.

2 The model

Our investigation in this work relies on version II of the 3-3-1 models [2]. Its lepton content comes in the fundamental representation of the $SU(3)_{\mathrm{L}}$, composing the following triplet:

$$f_{a\mathrm{L}} = \begin{pmatrix} \nu_a \\ e_a \\ \nu^{\mathrm{c}}_a \end{pmatrix}_{\mathrm{L}} \sim (1,\,3,\,-1/3), \;\; e_{a\mathrm{R}} \sim (1,1,-1), \quad (1)$$

with $a = 1,2,3$ representing the three known generations. We indicate the transformation under 3-3-1 after the similarity sign, "$\sim$".

In the quark sector, one generation comes in the triplet fundamental representation of $SU(3)_{\mathrm{L}}$ and the other two compose an anti-triplet with the following content:

$$Q_{i\mathrm{L}} = \begin{pmatrix} d_i \\ -u_i \\ d'_i \end{pmatrix}_{\mathrm{L}} \sim (3,\,\bar{3},\,0),$$

$$Q_{3\mathrm{L}} = \begin{pmatrix} u_3 \\ d_3 \\ u'_3 \end{pmatrix}_{\mathrm{L}} \sim (3,\,3,\,1/3),$$

$$u_{i\mathrm{R}} \sim (3,1,2/3),\; d_{i\mathrm{R}} \sim (3,1,-1/3),$$

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$$d'_{i\mathrm{R}} \sim (3,1,-1/3),$$
$$u_{3\mathrm{R}} \sim (3,1,2/3),\ d_{3\mathrm{R}} \sim (3,1,-1/3),$$
$$u'_{3\mathrm{R}} \sim (3,1,2/3), \tag{2}$$

where $a = 1,2,3$ and $j = 1,2$, both representing the different generations. The primed quarks are the exotic ones but with the usual electric charges.

In order to generate the correct mass for all massive particles, the model requires only three triplets of scalars, namely

$$\eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta'^0 \end{pmatrix},\ \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho'^+ \end{pmatrix},\ \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix}, \tag{3}$$

with η and χ both transforming as (1, 3, $-1/3$) and ρ transforming as (1, 3, 2/3).

In the gauge sector, the model recovers the standard gauge bosons and disposes of five more other ones, called $V^\pm$, U^0, $U^{0\dagger}$ and Z^2 [2].

Many of these new particles are bileptons (carrying two units of lepton number) [10]

$$L(V^+,\ U^{\dagger 0},\ u'_3,\ \eta'^0,\ \rho'^+) = -2,$$
$$L(V^-,\ U^0,\ d'_i,\ \chi^0,\ \chi^-) = +2. \tag{4}$$

There are two things that deserve attention in this lepton number distribution above. First, notice that the new quarks, u'_3 and d'_i are leptoquarks once they carry lepton and baryon numbers. Second, we have two neutral scalars bileptons, η'^0 and χ^0. Therefore when one or both of these neutral scalar bileptons develop a vacuum expectation value (VEV), we are going to have spontaneous breaking of the lepton number.

In what concerns the potential, it is suitable to note that if we impose lepton number conservation and assume the discrete symmetry $\chi \to -\chi$, the potential we can form with the three scalar triplets above,

$$\begin{aligned} &V(\eta,\rho,\chi) \qquad\qquad\qquad (5)\\ &= \mu_\chi^2\chi^2 + \mu_\eta^2\eta^2 + \mu_\rho^2\rho^2 + \lambda_1\chi^4 + \lambda_2\eta^4 + \lambda_3\rho^4 \\ &\quad +\lambda_4(\chi^\dagger\chi)(\eta^\dagger\eta) + \lambda_5(\chi^\dagger\chi)(\rho^\dagger\rho) + \lambda_6(\eta^\dagger\eta)(\rho^\dagger\rho) \\ &\quad +\lambda_7(\chi^\dagger\eta)(\eta^\dagger\chi) + \lambda_8(\chi^\dagger\rho)(\rho^\dagger\chi) + \lambda_9(\eta^\dagger\rho)(\rho^\dagger\eta), \end{aligned}$$

has the striking feature of providing an extra global symmetry $U(1)$ with the three triplets of scalars transforming in the following way by this symmetry: η, ρ, $\chi \sim (1)$. The symmetry can be extended to the entire Lagrangian turning it into a symmetry of the model [7]. To accomplish this the multiplets of matter must transform as $Q_{1\mathrm{L}} \sim (1)$, $Q_{i\mathrm{L}} \sim (-1)$, $f_{a\mathrm{L}} \sim (-1/2)$ and $e_{a\mathrm{R}} \sim (-3/2)$ under the new $U(1)$, with all other multiplets not transforming at all. The advantage of having this extra symmetry is that it can be identified with the PQ symmetry [6], which might potentially provide a solution to the strong CP problem in the context of the 3-3-1 model[1]. This realization of PQ symmetry in 3-3-1 was first observed by Pal in [7]. Pal also recognized that such a scenario was not realistic once the spontaneous breaking of this PQ symmetry implied a Weinberg–Wilczek axion type [11], already ruled out phenomenologically. What is interesting here is the fact that the PQ symmetry is automatic in the minimal model.

Despite of the fact that the PQ symmetry in this minimal scenario is useless, it was shown in [10] that in order to render the PQ symmetry useful we just have to add a scalar singlet to the minimal content of the model. Then with the price of introducing a scalar singlet we have a solution to the strong CP problem.

In this paper we will not consider the extension made in [10] that turned the PQ symmetry into a viable symmetry; instead we consider a term in the potential above that breaks explicitly the PQ symmetry, but maintains the lepton number symmetry. For this we have to discard the discrete symmetry $\chi \to -\chi$ took above. In demanding lepton number conservation, the only possible term that we can add to the potential above is this:

$$\frac{f}{\sqrt{2}}\epsilon^{ijk}\eta_i\rho_j\chi_k, \tag{6}$$

which explicitly breaks the PQ symmetry. This term is expected to yield a mass for the axion around the scale of v_η which is of the order of a few hundreds of GeV.

In summary, version II of the 3-3-1 models presents two global symmetries, namely the PQ and the lepton number symmetry. Of these symmetries, only the former was already developed, as discussed above. The contribution of this work to the development of this version of the 3-3-1 models is to complete the study of their global symmetries by implementing the spontaneous breaking of the lepton number symmetry.

The idea in the next section is to break spontaneously the lepton number. As we will see next, we do not need to add anything else to the model in order to implement the breaking of the lepton number once the scalar content of the model disposes of two neutral scalars bileptons, namely η'^0 and χ^0. What we have to do is to allow one or both of these scalar bileptons to develop a VEV. For the sake of simplicity let us develop the case where only η'^0 develops a VEV.

3 Spontaneously broken lepton number

Before we go on, it is important to stress that the model we are treating here was built in such way that lepton number is a symmetry of the Lagrangian. The existence of this global symmetry forbids neutrinos of having a Majorana

[1] Notice that there is an interdependence between PQ symmetry and lepton symmetry in the sense that the discrete symmetry $\chi \to -\chi$ alone is not sufficient to avoid trilinear terms like $\eta\eta\rho$ and $\chi\chi\rho$ which violate the PQ symmetry. These trilinear terms are absent only when lepton number conservation is imposed.

mass term. As there is evidence that neutrinos are massive [12] and possibly Majorana-like [13], there is also a strong motivation to push for lepton number violation. In this section we implement the spontaneous breaking of the lepton number. The case is interesting because a Goldstone boson appears in the spectrum which is called a majoron. As such a majoron comes from a multiplet, it can have appealing cosmological and astrophysical consequences [14].

We start expanding η'^0, η^0, ρ^0 and $\chi^{0\prime}$ around its VEV, $v_{\eta',\eta,\rho,\chi'}$, in the usual way,

$$\eta'^0, \eta^0, \rho^0, \chi'^0 \to \frac{1}{\sqrt{2}}(v_{\eta',\eta,\rho,\chi'} + R_{\eta',\eta,\rho,\chi'} + \mathrm{i}I_{\eta',\eta,\rho,\chi'}). \tag{7}$$

On substituting this expansion in the potential formed with (5) and (6), we obtain the following set of constraints:

$$\begin{aligned}
&\mu_\chi^2 + \lambda_1 v_{\chi'}^2 + \frac{\lambda_4}{2}(v_\eta^2 + v_{\eta'}^2) + \frac{\lambda_5}{2} v_\rho^2 \\
&\quad + \frac{\lambda_7}{2} v_{\eta'}^2 + \frac{f}{2}\frac{v_\eta v_\rho}{v_{\chi'}} = 0, \qquad (8)\\
&\mu_\eta^2 + \lambda_2(v_{\eta'}^2 + v_\eta^2) + \frac{\lambda_4}{2} v_{\chi'}^2 + \frac{\lambda_6}{2} v_\rho^2 + \frac{\lambda_7}{2} v_{\chi'}^2 = 0, \\
&\mu_\eta^2 + \lambda_2(v_{\eta'}^2 + v_\eta^2) + \frac{\lambda_4}{2} v_{\chi'}^2 + \frac{\lambda_6}{2} v_\rho^2 + \frac{f}{2}\frac{v_\rho v_{\chi'}}{v_\eta} = 0, \\
&\mu_\rho^2 + \lambda_3 v_\rho^2 + \frac{\lambda_5}{2} v_{\chi'}^2 + \frac{\lambda_6}{2}(v_{\eta'}^2 + v_\eta^2) + \frac{f}{2}\frac{v_\eta v_{\chi'}}{v_\rho} = 0.
\end{aligned}$$

Notice that the second and third constraints imply the relation

$$\lambda_7 v_{\chi'}^2 - f\frac{v_\rho v_{\chi'}}{v_\eta} = 0, \tag{9}$$

which avoids the presence of dangerous tadpoles with $R_{\chi'}$, stemming from the terms $\lambda_7(\chi^\dagger\eta)(\eta^\dagger\chi)$ and $\frac{f}{\sqrt{2}}\epsilon^{ijk}\eta_i\rho_j\chi_k$ in the potential.

With these constraints, the potential formed with (5) and (6) leads to the following mass matrix M_{R}^2 for the neutral CP-even scalars in the basis $(R_\chi, R_{\eta'}, R_{\chi'}, R_\eta, R_\rho)$:

$$\left(\begin{array}{ccc}
-\frac{\lambda_7 v_{\eta'}^2}{4} & 0 & \frac{\lambda_7 v_\eta v_{\eta'}}{4} \\
0 & \lambda_2 v_{\eta'}^2 & \frac{1}{2}(\lambda_4+\lambda_7)v_{\chi'}v_{\eta'} \\
\frac{\lambda_7 v_\eta v_{\eta'}}{4} & \frac{1}{2}(\lambda_4+\lambda_7)v_{\chi'}v_{\eta'} & \lambda_1 v_{\chi'}^2 \\
\frac{\lambda_7 v_{\chi'} v_{\eta'}}{4} & \lambda_2 v_\eta v_{\eta'} & \frac{\lambda_4 v_{\chi'} v_\eta}{2} + \frac{\lambda_7 v_{\chi'} v_\eta}{4} \\
-\frac{\lambda_7}{4}\frac{v_\eta v_{\chi'} v_{\eta'}}{v_\rho} & \frac{\lambda_6}{2} v_{\eta'} v_\rho & \frac{\lambda_5 v_{\chi'} v_\rho}{2} + \frac{\lambda_7}{4}\frac{v_{\chi'} v_\eta^2}{v_\rho}
\end{array}\right.$$

$$\left.\begin{array}{cc}
\frac{\lambda_7 v_{\chi'} v_{\eta'}}{4} & -\frac{\lambda_7}{4}\frac{v_\eta v_{\chi'} v_{\eta'}}{v_\rho} \\
\lambda_2 v_\eta v_{\eta'} & \frac{\lambda_6}{2} v_{\eta'} v_\rho \\
\frac{\lambda_4 v_{\chi'} v_\eta}{2} + \frac{\lambda_7 v_{\chi'} v_\eta}{4} & \frac{\lambda_5 v_{\chi'} v_\rho}{2} + \frac{\lambda_7}{4}\frac{v_{\chi'} v_\eta^2}{v_\rho} \\
\lambda_2 v_\eta^2 - \frac{\lambda_7 v_{\chi'}^2}{4} & \frac{\lambda_6 v_\eta v_\rho}{2} + \frac{\lambda_7}{4}\frac{v_{\chi'}^2 v_\eta}{v_\rho} \\
\frac{\lambda_6 v_\eta v_\rho}{2} + \frac{\lambda_7}{4}\frac{v_{\chi'}^2 v_\eta}{v_\rho} & \lambda_3 v_\rho^2 - \frac{\lambda_7}{4}\frac{v_{\chi'}^2 v_\eta^2}{v_\rho^2}
\end{array}\right). \tag{10}$$

Although the diagonalization of this matrix can be extremely tough, one can straightforwardly check that it yields a null eigenvalue by writing the secular equation for its determinant. This information is all that we need to detect the number of Goldstone bosons among these real scalars.

For the pseudo-scalars we have the following mass matrix M_I^2 in the basis $(I_{\eta'}, I_\chi, I_{\chi'}, I_\eta, I_\rho)$:

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & -\frac{\lambda_7 v_{\eta'}^2}{4} & \frac{\lambda_7 v_\eta v_{\eta'}}{4} & \frac{\lambda_7 v_{\eta'} v_{\chi'}}{4} & \frac{\lambda_7}{4}\frac{v_{\chi'} v_{\eta'} v_\eta}{v_\rho} \\
0 & \frac{\lambda_7 v_\eta v_{\eta'}}{4} & -\frac{\lambda_7 v_\eta^2}{4} & -\frac{\lambda_7 v_{\chi'} v_\eta}{4} & -\frac{\lambda_7}{4}\frac{v_{\chi'} v_\eta^2}{v_\rho} \\
0 & \frac{\lambda_7 v_{\chi'} v_{\eta'}}{4} & -\frac{\lambda_7 v_{\chi'} v_\eta}{4} & -\frac{\lambda_7 v_{\chi'}^2}{4} & -\frac{\lambda_7}{4}\frac{v_{\chi'}^2 v_\eta}{v_\rho} \\
0 & \frac{\lambda_7}{4}\frac{v_{\chi'} v_{\eta'} v_\eta}{v_\rho} & -\frac{\lambda_7}{4}\frac{v_{\chi'} v_\eta^2}{v_\rho} & -\frac{\lambda_7}{4}\frac{v_{\chi'}^2 v_\eta}{v_\rho} & -\frac{\lambda_7 v_\eta^2 v_{\chi'}^2}{4 v_\rho^2}
\end{pmatrix}. \tag{11}$$

From this mass matrix we can easily see that the $I_{\eta'}$ remains massless and decouples from the other pseudo-scalars, I_χ, $I_{\chi'}$, I_η, I_ρ, which, after diagonalization, combine among themselves to generate the Goldstone bosons. These, along with the CP-even Goldstone boson obtained from diagonalization of M_{R}^2, form the set of Goldstone bosons that will be eaten by the massive neutral gauge bosons of the model. We then end up with an extra massless pseudo-scalar, $J = I_{\eta'}$, which is decoupled from the other scalars. This massless pseudo-scalar is the result of the spontaneous breaking of the lepton number. In the literature this pseudo-scalar is the so called majoron.

It is opportune to remark that, although we have gotten the right number of Goldstones and the majoron, we have not explicitly shown that the true vacuum is the one that we assumed here. As was pointed out some years ago [15], it is necessary to analyze the possibility of having a broken phase which does not correspond to a minimum of the potential. In other words, we have to be sure that our solution leads to a minimum and not a saddle point. In our case, it would be enough to guarantee that the mass matrices, (10) and (11), lead to positive eigenvalues, since they would correspond to positive second derivatives of the potential with respect to the fields at the minimum. We have checked that for values of the λ's of the order of 0.1 (and $\lambda_7 < 0$) and the VEV's $v_\eta = v_\rho \approx 100\,\mathrm{GeV}$, $v_{\eta'} \approx 1\,\mathrm{MeV}$ and $v_\chi \approx 1\,\mathrm{TeV}$, we obtained that all the mass eigenvalues are positive and above 1 TeV, except for the majoron partner, which has a mass about 3 eV, consistent with what we expect phenomenologically. Then we are safely talking about a true vacuum which corresponds to a minimum of the potential in this case.

Finally, it is important to remember that after the spontaneous breaking of the 3-3-1 symmetry the triplet η in (3) dissociates into a doublet $(\eta^0\ \ \eta^-)^{\mathrm{T}}$ plus a singlet η'^0 by the standard $SU(3)_C \times SU(2)_{\mathrm{L}} \times U(1)_Y$ (3-2-1) symmetry. As we saw, our majoron comes from η'^0; thus it is a singlet by the 3-2-1 symmetry. We know that a singlet majoron in the SM is trivially invisible because it interacts at tree level only with right-handed neutrinos. We will show in Sect. 4 that this is not the case here. Even though our majoron is a singlet by the 3-2-1 symmetry, we cannot automatically jump to the conclusion that it is an invisible one because it originates from a triplet by the 3-3-1 symmetry; therefore it interacts with the neutral gauge boson Z_1, which plays the role of the standard neutral gauge bosons. In this case what threatens the invisibility of our majoron is the decay $Z_1 \to R_{\eta'} + I_{\eta'}$. Besides, it couples to the charged gauge bosons of the model, $W^\pm$ and $V^\pm$ [2], which leads to an effective coupling to charged leptons and we have to care about its size, which we do next.

4 Majoron phenomenology

As we saw, the majoron is a massless pseudo-scalar originating from the spontaneous breaking of the lepton number global symmetry. This is possible in extensions of the SM possessing at least an additional multiplet of scalars. In such extensions, majorons that belong to either a triplet [17] or a doublet [18] can interact with neutrinos and charged leptons, being severely constrained by astrophysical, cosmological [14] and laboratory data [19]. The reason that such phenomenological data disfavor the usual multiplet majorons is related to the Z_0 invisible decay rate [19]. The problem comes from astrophysical bounds on the VEV that breaks the lepton symmetry. Basically, these bounds demand that triplet or doublet majoron scenarios develop a VEV around keV in order to avoid too fast cooling of red giants [14]. This constraint is derived through Compton scattering to a majoron, $\gamma + e \to e + J$. Then, the neutral real scalar partner of the majoron, let us call it R_J, receives a mass of the order of keV and therefore contributes to the Z^0 invisible decay $Z^0 \to R_J J$ [19]. Nonetheless, such a decay is not allowed for a light R_J since the measured invisible Z_0 decay rate is well explained by three neutrinos and this extra contribution is adding up to increase this rate by an unacceptable amount. For this reason a successful majoron model is expected to have its origin in a singlet by the standard 3-2-1 symmetry. A singlet majoron model was first suggested by Chikashige, Mohapatra and Peccei (CMP) in [16]. In view of this, a criterion to decide if a majoron emerging from extensions of the SM can be established by demanding that the scalar that gives rise to the majoron be a singlet by the 3-2-1 symmetry. Unfortunately this criterion cannot be used for our majoron.

Despite that our majoron is a singlet by the 3-2-1 symmetry, it presents some differences from the CMP majoron. Namely, our majoron interacts with all the gauge bosons of the model, particularly the Z^1, which is the equivalent of the standard neutral gauge boson Z^0. Moreover, on evaluating numerically the matrix M_{R}^2 in (10) for typical values for the parameters involved in it, we will find that the mass of R_J is proportional to $v_{\eta'}$. In face of this, it is recommended that we check if for small $v_{\eta'}$ the decay $Z^1 \to R_J J$ does not rule out our scenario.

After the symmetry breaking from 3-3-1 to the $SU(3)_C \times U(1)_{\mathrm{em}}$, the Z^0 get mixed with the Z' and form the physical neutral gauge bosons $Z^1 = Z^0 C_\theta - Z' S_\theta$ and $Z^2 = Z^0 S_\theta + Z' C_\theta$ [20]. Our interest here lies upon Z^1 because it will play the role of the standard neutral gauge boson [20]. With this mixing we obtain, from the Higgs boson kinetic term $(D_\mu \eta)^\dagger (D^\mu \eta)$, the following interaction among the majoron and the neutral gauge boson Z^1:

$$\mathcal{L}_{Z^1 R_J J} = -\frac{1}{3} g \sqrt{3+t^2} S_\theta \left(\partial^\mu R_J J - \partial^\mu J R_J\right) Z^1_\mu, \quad (12)$$

where g is the coupling constant for the weak-isospin group $SU(3)_{\mathrm{L}}$ which coincides with the standard one [2], $t = \frac{\sqrt{3} S_{\mathrm{W}}}{\sqrt{3-4S_{\mathrm{W}}^2}}$ and $S_{\mathrm{W}} = \sin\theta_{\mathrm{W}}$ with θ_{W} being the Weinberg angle. This interaction leads to the following expression for the decay rate $Z^1 \to R_J\ J$:

$$\Gamma_{Z^1 \to R_J J} = \frac{4}{9} g^2 (3+t^2) S_\theta^2 m_{Z^1} = \frac{2}{3} S_\theta^2 C_{\mathrm{W}}^2 (3+t^2) \Gamma_{\nu\nu}, \quad (13)$$

where $\Gamma_{\nu\nu} = \frac{G_{\mathrm{F}} m_{Z^1}^3}{12\sqrt{2}\pi}$ is the prediction for the decay rate of Z^1 into a pair of neutrinos. For $S_{\mathrm{W}}^2 = 0.23$, which yields $t = 0.57$, we obtain

$$\Gamma_{Z^1 \to R_J J} = 1.66 S_\theta^2 \Gamma_{\nu\nu}. \quad (14)$$

From the experimental side, we have [21]

$$\Gamma_{\mathrm{inv}}^{\mathrm{ex}} = (2.993 \pm 0.011) \Gamma_{\nu\nu}. \quad (15)$$

Assuming that there are only three species of neutrinos, the window for new physics concerning the invisible decay of Z^1 is

$$\Gamma_{\mathrm{inv}}^{\mathrm{NP}} \leq 0.004 \Gamma_{\nu\nu}. \quad (16)$$

From (16) and (14) we obtain the constraint

$$S_\theta \leq 0.049. \quad (17)$$

There is an upper bound on this angle: $\theta \leq 0.000132$ [20]. As long as this upper bound is obeyed we can safely say that the decay $Z^1 \to R_J J$ does not rule out our majoron.

Another source of phenomenological constraints against the existence of the majoron arises from its coupling to matter, particularly to the electron. The interaction among any lepton and the majoron only appears through loop corrections. In regard to the electron, the main contribution to such an interaction is depicted in Fig. 1, which originates from the Lagrangian

$$\mathcal{L} = \frac{g}{\sqrt{2}} \left(\bar{\nu}_{\mathrm{L}} \gamma^\mu e_{\mathrm{L}}^- W_\mu^+ + \bar{\nu}_{\mathrm{L}}^{\mathrm{C}} \gamma^\mu e_{\mathrm{L}}^- V_\mu^+\right)$$

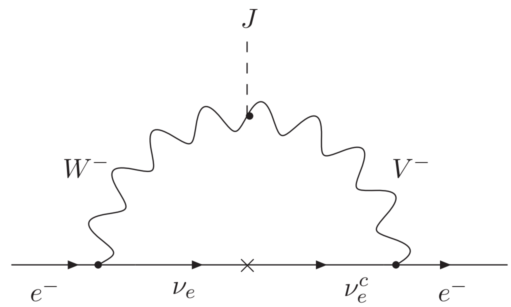

Fig. 1. Main contribution to g_{eeJ} coupling

$$+\frac{g^2}{2} v_\eta W^+ V^- J + \text{H.c.} \tag{18}$$

With these interactions we obtain the following approximate expression for the electron–electron–majoron coupling:

$$g_{eeJ} \simeq \frac{g^4 m_{\nu_e}^2 m_e v_\eta}{16\pi^2 m_W^2 m_V^2}. \tag{19}$$

The experimental constraint is $g_{eeJ} < 10^{-18}$ [22], which is obviously satisfied by typical values of the parameters involved in (19).

Once we are sure that our majoron is safe from phenomenological constraints, we can go a step further and present what is the main signal of our majoron. In accordance with the Yukawa interactions of this model [2], the triplet η only interacts with quarks. This means that our majoron does not interact directly with any lepton. Particularly our majoron interactions with fermions involve only the leptoquark and the ordinary quark [2]. In view of this we would expect that its main signal is the decay of a leptoquark in an ordinary quark plus majoron, $q' \to q + J$. The decay rate in this case is

$$\Gamma(q' \to q + J) = \frac{h^2 m_q}{16\pi}. \tag{20}$$

In this rate m_q stands for the mass of an ordinary quark, with h representing the Yukawa strength interactions among leptoquark–quark–majoron. This means that the discovery of this majoron depends on the existence of the exotic quarks u' and d', which are characteristic of the 3-3-1 models.

Let us establish a classification for our majoron. We saw that as consequences of the spontaneous breaking of the lepton number a majoron showed up in the model. In the literature majorons are classified as singlet or multiplet majorons. This classification is based on majorons that come from extensions of the standard model. In view of this, the common majorons are singlet [16], doublet [17] or triplet [18] majorons. Phenomenological constraints have ruled out the doublet and the triplet majorons [19], allowing for the singlet majoron only. We have a peculiar situation here. Our majoron is a triplet by the 3-3-1 symmetry, but it is a singlet by the 3-2-1 symmetry. However, it interacts with fermions and gauge bosons, which is typical of multiplet majorons (double or triplet). It is due to this, and to the fact that our majoron has its origin in a triplet by the 3-3-1 symmetry, that we decided to classify our majoron as a multiplet majoron, particularly a triplet majoron.

We finish this section pointing out that our majoron presents the peculiar feature of interacting with quarks instead of interacting with leptons, exactly the contrary to the other multiplet majorons. It is this fact that turns our majoron phenomenologically distinct from the other multiplet majorons.

5 Upper bound on $v_{\eta'}$ and neutrino masses

As the majoron has its origin in a triplet, its associated vacuum $v_{\eta'}$ should contribute to the ρ parameter. In order to check this, let us obtain the expression for ρ. The definition of ρ here goes like in the SM: $\rho = \frac{m_{W^+}^2}{m_{Z_1}^2 C_{\mathrm{W}}^2}$. The respective expressions for $m_{W^+}^2$ and $m_{Z_1}^2$ in leading order in $\frac{1}{v_{\chi'}}$ are

$$\begin{aligned} m_{W^+}^2 &= \frac{g^2}{4}\left(v_\rho^2 + v_\eta^2 - \frac{v_\eta^2 v_{\eta'}^2}{v_{\chi'}^2}\right), \\ m_{Z_1}^2 &= \frac{g^2(v_\rho^2 + v_\eta^2)}{4C_W^2} \\ &\times \left(1 - \frac{3+4t^2}{108}\left(\frac{5(v_\rho^2+v_\eta^2)}{12 v_{\chi'}^2} + \frac{9+56\sqrt{2}}{\sqrt{2}}\frac{v_{\eta'}^2}{v_{\chi'}^2}\right)\right). \end{aligned} \tag{21}$$

Observe that if we take $v_{\eta'} = 0$, we recover the masses predicted by the original version of the model [2].

With these masses we obtain the following expression for ρ:

$$\rho = \frac{v_\rho^2 + v_\eta^2 - \frac{v_\eta^2 v_{\eta'}^2}{v_\chi^2}}{(v_\rho^2 + v_\eta^2)\left(1 - \frac{3+4t^2}{108}\left(\frac{5(v_\rho^2+v_\eta^2)}{12v_\chi^2} + \frac{9+56\sqrt{2}}{\sqrt{2}}\frac{v_{\eta'}^2}{v_\chi^2}\right)\right)}. \tag{22}$$

The present value for ρ is $\rho = 1.0012^{+0.0023}_{-0.0014}$. Taking appropriate values for the parameters in (22) ($v_\rho = v_\eta = 10^2$ GeV and $v_\chi = 10^3$ GeV), we obtain the following upper bound upon $v_{\eta'}$:

$$v_{\eta'} \leq 40 \text{ GeV}. \tag{23}$$

The parameter ρ is very sensitive to the value of v_χ. For example, for $v_\chi = 500\,\text{GeV}$ we get $v_{\eta'} \leq 16\,\text{GeV}$.

Let us now show that the breaking of the lepton number through $v_{\eta'} \neq 0$ engenders Majorana mass for the neutrinos. One can see this by noticing that when η'^0 develops a VEV, the Yukawa interaction $h_{ab}\bar{f}_{a\mathrm{L}} e_{b\mathrm{R}} \rho$ [2], together with the term $\lambda_9(\eta^\dagger\rho)(\rho^\dagger\eta)$ in the potential, generate Majorana neutrino mass through one loop as depicted in Fig. 2. In a naive approximation such a loop provides the following Majorana mass terms for the neutrinos:

$$m_{ij}^\nu \approx \frac{\lambda_9 h_{ia} m_a h_{aj} v_\eta v_{\eta'}}{m_{\rho'^+}^2}, \tag{24}$$

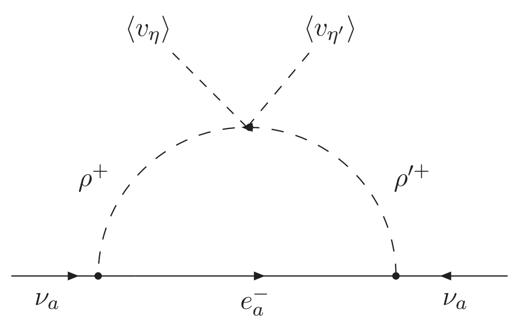

Fig. 2. One loop diagram that leads to Majorana neutrino mass

with $i, j = 1, 2, 3$ and $a = e, \mu, \tau$.

On the other side the Yukawa interaction $h_{ab}\bar{f}_{a\mathrm{L}}e_{b\mathrm{R}}\rho$ generates the following charged lepton mass matrix: $m^l_{ab} = h_{ab}v_\rho$. In principle the model has the stage settled to compute masses for all leptons if we can extract information about the matrix elements h_{ab}. This subject deserves careful analysis since only appropriate patterns of mixing could reproduce the recent data on neutrino physics. Although we will study this in detail elsewhere [23], here we can at least check if the matrix elements in (24) can be obtained within a reasonable order of magnitude considering neutrinos acquire masses around eV's. To accomplish this we make some assumptions concerning the values of the parameters involved in lepton masses. Namely, $v_\rho \approx 10^2$ GeV (responsible for charged lepton masses), $v_\eta \approx 10^2$ GeV (the electro-weak scale), $v_{\eta'} \approx 1$ MeV (it could be even of the order of keV's since lepton number is only very softly broken), $m_{\rho^{+\prime}} \approx 10^3$ GeV (since it is a typical scalar related to 3-3-1 symmetry) and $\lambda_9 \approx 1$. If we take the largest matrix element h_{ij}, that related to the tau mass, we see that it has to be of the order of 10^{-2}; this amounts to

$$m^\nu_{ij} \approx 10^{-2}\,\mathrm{eV}, \tag{25}$$

which is an impressive value for the order of magnitude for a neutrino mass. It will be a great achievement for this 3-3-1 model if besides providing an invisible majoron the correct pattern of neutrino mixing and masses emerge naturally from the model.

6 Conclusions

The contribution of this work to the development of version II of the 3-3-1 models is the implementation of the spontaneous breaking of the lepton number. The importance of this is the fact that lepton number violation is a necessary condition to generate Majorana neutrino mass. In view of this the main result of this paper is that spontaneously broken lepton number is viable once the majoron is invisible. It is important to remember that we achieved this without any modification of the minimal model; basically we just allowed η'^0 to develop a VEV.

To finalize, in general the 3-3-1 models present two extra global symmetries, namely, the PQ and the lepton number symmetries. Moreover their scalar sector provides a simple implementation of the spontaneous breaking of such symmetries [7, 8]. Regarding the PQ symmetry, it was shown in [7] that in both versions the spontaneous breaking of the PQ symmetry imply a Weinberg–Wilczek axion type [11] already ruled out phenomenologically, turning then such symmetry useless. Recently it was shown in [10] that in version II the PQ symmetry regains its usefulness by the addition of a simple scalar singlet. In regard to the lepton number symmetry, it was shown in [8] that the majoron that comes from the spontaneous breaking of the lepton number in version I is identical to the Gelmini–Roncadeli one [18], which is already ruled out phenomenologically. In this work we completed this sequence of investigations by showing that in version II the spontaneous breaking of the lepton number implied an invisible majoron. Such a result puts version II in a privileged position. Moreover, as we saw in Sect. 5, it seems that the model has all the ingredients to provide the correct neutrino masses. In this sense, it would be possible to have two strong candidates for cold dark matter and simultaneously solve the neutrino puzzle along with the strong-CP problem [23].

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